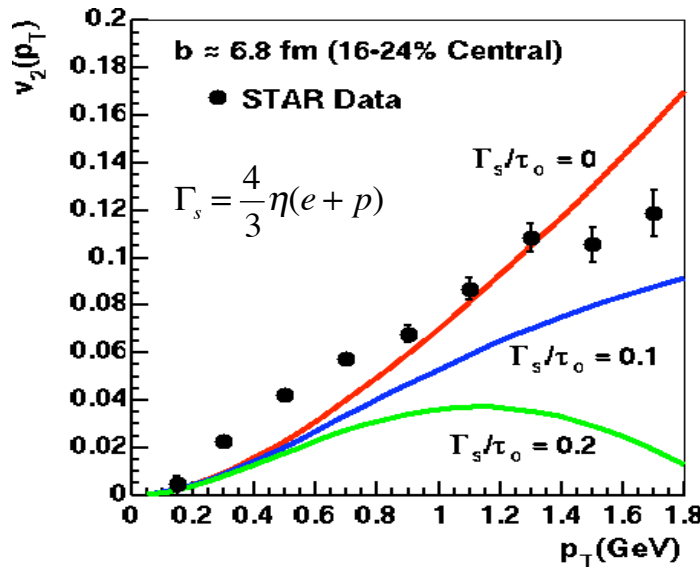


Global fits of v_2 and their implications to Hydrodynamical limit and η/s

Aihong Tang, Raimond Snellings and Hiroshi Masui

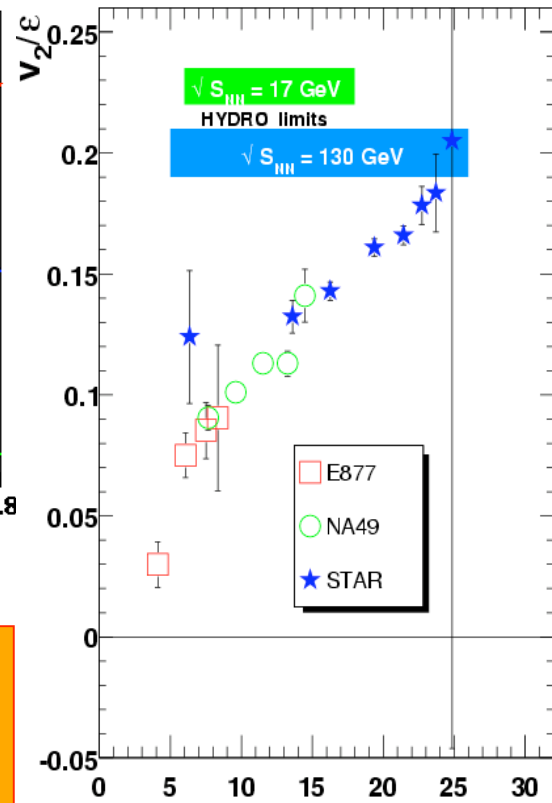


The story rewinded

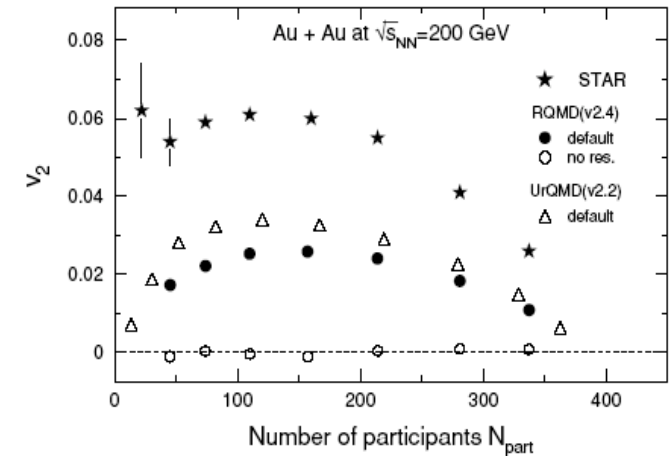


D. Teaney, PRC 68 034913 (2003)

Viscosity reduces v_2
Viscosity needs to be small
in order to explain data



STAR, PRC 66 034904 (2002)

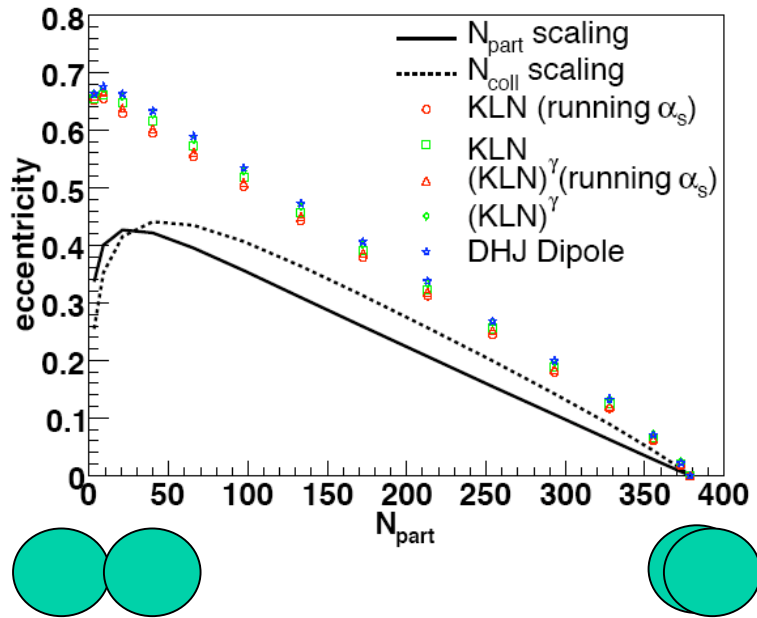


Y. Lu et al. Journal of Phys. G 32 1121 (2006)

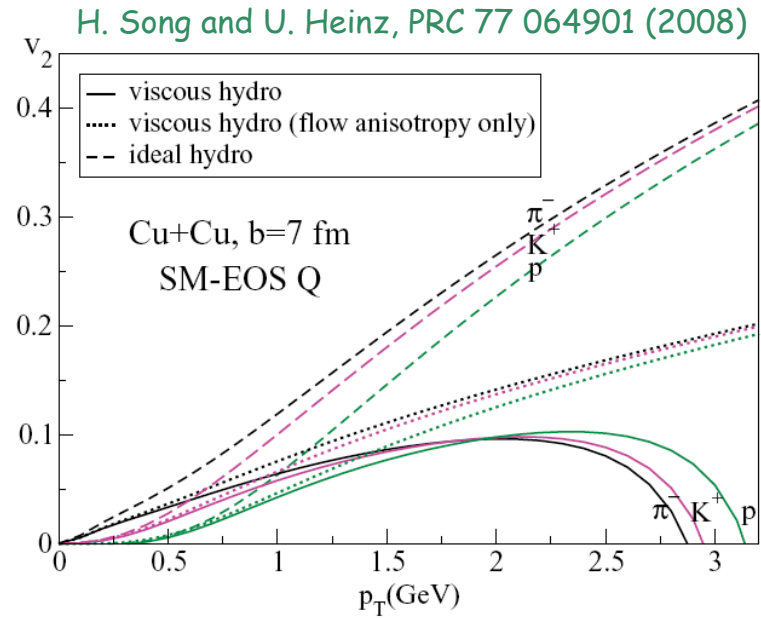
v_2/ϵ approaches the limit
of ideal hydrodynamics
Hadronic interaction
alone does not produce
enough v_2



The story continues



T. Hirano, RHIC & AGS Users Mtg 06

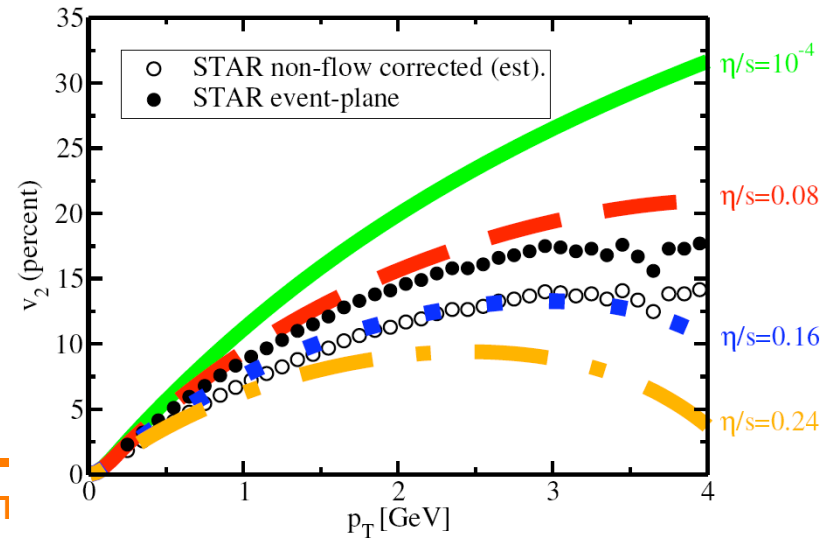
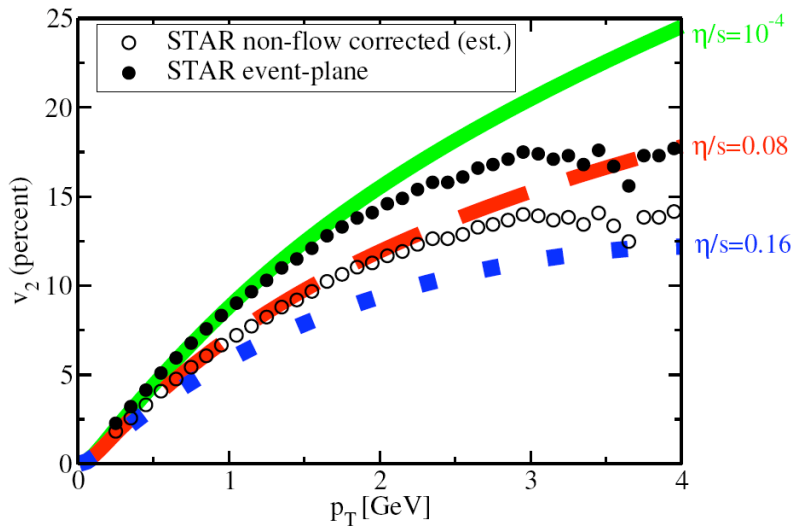


H. Song and U. Heinz, PRC 77 064901 (2008)

Glauber

M. Luzum and P. Romatschke arXiv0804.4015

CGC



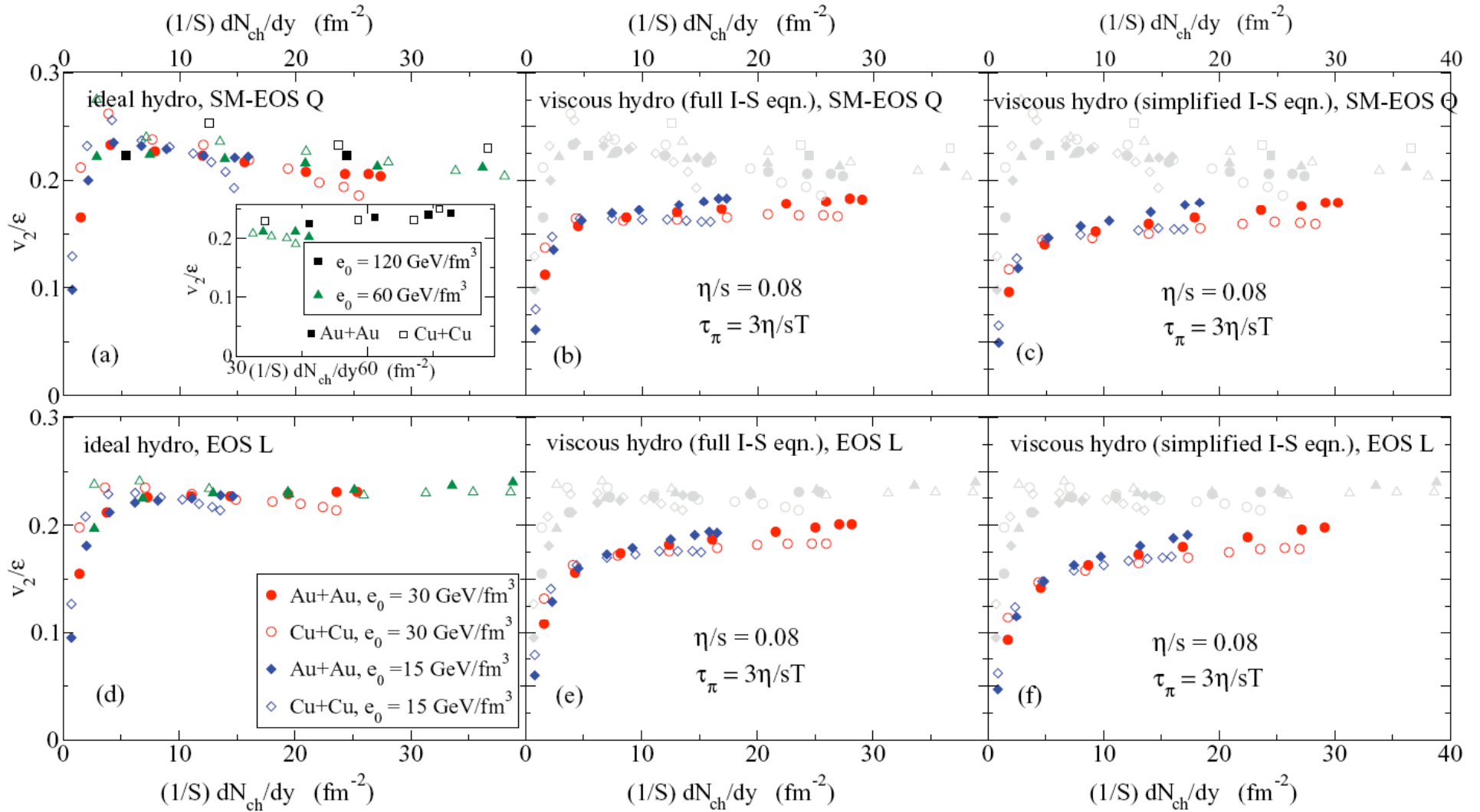
ing 1

and others, Tsuku



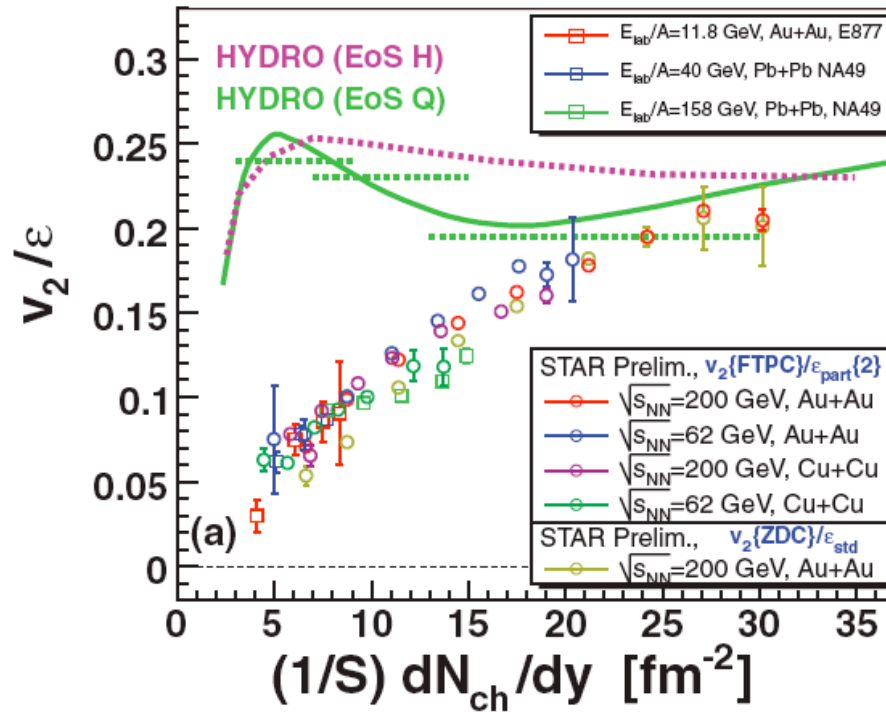
The story continues

H. Song and U. Heinz, PRC 78 024902 (2008)

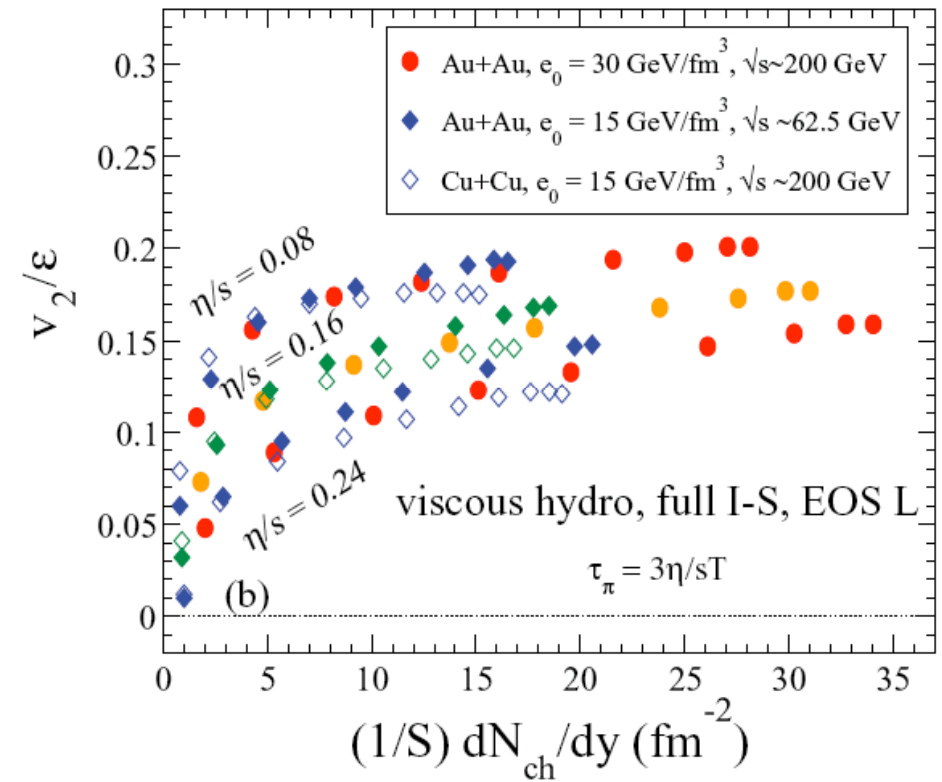




The story continues



STAR, PRC 66 034904 (2002)
S.Voloshin, AIP Conf. Proc. 870, 691 (2006)



H. Song and U. Heinz, PRC 78 024902 (2008)



How to view the hydro behavior better ? - Move away from it



- Ideal fluid and low viscosity \Leftrightarrow local equilibrium (small λ or large σ)

- **To study the local equilibrium, we have to move away from it,** say, check what if we relax the constraint of local equilibrium

- How to get a complete view? Study Boltzman equation for diluted system. It recovers Hydro when λ becomes small.

“To have a complete view of Lu Mountain, one has to move away from it.”

- Shi Su (1037~1101)



Transport Theory and Hydrodynamics

Transport Theory	Hydrodynamics
Microscopic	Macroscopic
Applicable out of equilibrium	Local equilibrium
Cannot describe phase transition	Can treat phase transition
$D \ll 1$	$K \ll 1$

D (Dilution parameter) =

$$\frac{\text{Typical distance between two particles}}{\text{Mean free path}}$$

K (Knudsen number) =

$$\frac{\text{Mean free path}}{\text{System size}}$$

Boltzmann Equation will be reduced to Hydrodynamics when both $D \ll 1$ and $K \ll 1$



Connecting Pieces

$$D \equiv \frac{n^{-1/3}}{\lambda} = \sigma n^{2/3}$$

n : particle density
 σ : parton cross section
 R : system size
 λ : mean free path

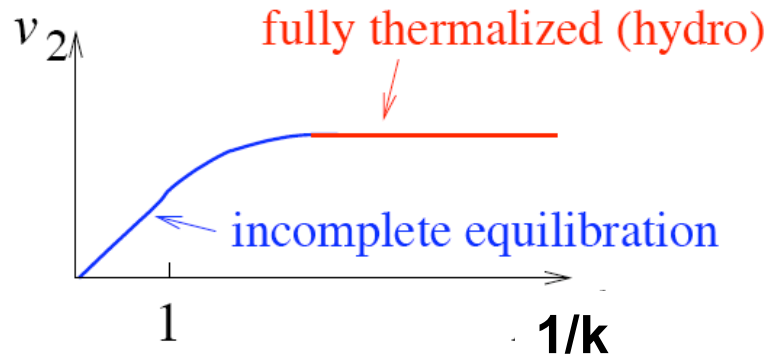
$$\frac{1}{K} \equiv \frac{R}{\lambda} \quad \leftarrow \text{Number of collisions.}$$

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct} \frac{1}{S} \frac{dN}{dy}$$

$$t \sim R / c_s$$

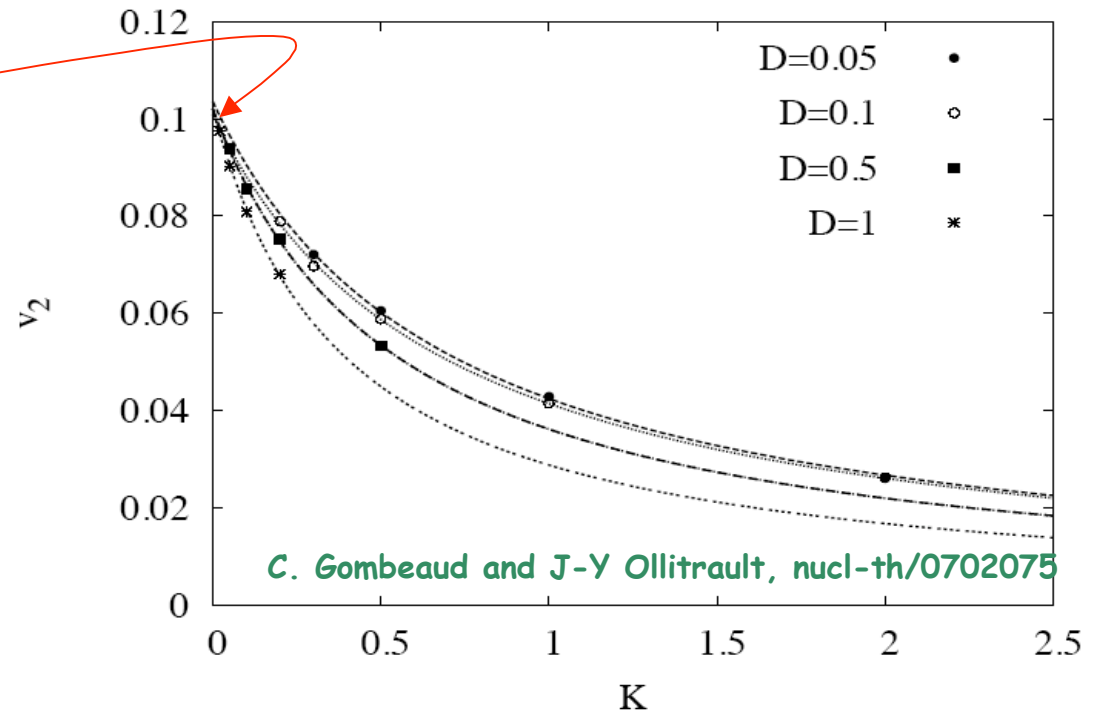
$$\frac{1}{K} = \sigma \left[\frac{1}{S} \frac{dN}{dy} \right] \frac{c_s}{c}$$





v_2 from Solving the Boltzmann Equation

Hydro limit is recovered
when $D \ll 1$ and $K \ll 1$



$v_2/\varepsilon \propto 1/K$, when K is large
(low density limit)
 $v_2/\varepsilon \propto K$, when K is small
(ideal Hydro limit)

\Rightarrow

$$\frac{v_2}{\varepsilon} = \left[\frac{v_2}{\varepsilon} \right]_{hydro} \frac{1}{1 + K / K_0}$$

$$\frac{v_2}{\varepsilon} = \left[\frac{v_2}{\varepsilon} \right]_{hydro} \frac{2}{\pi} \text{atan} \left(\frac{1}{K / K_0} \right)$$

$$\frac{v_2}{\varepsilon} = \left[\frac{v_2}{\varepsilon} \right]_{hydro} \frac{1}{2} \left(1 - e^{-\frac{1}{K/K_0}} + e^{-K/K_0} \right)$$



Choose the right $\{v_2, \varepsilon\}$ pairs

v_2 that are sensitive to anisotropy w.r.t. the **Reaction Plane v_2** :

$v_2\{4\}$, $v_2\{qDist\}$,
 $v_2\{qCumulant4\}$, $v_2\{ZDCSMD\}$

ε that are sensitive to anisotropy w.r.t. the **Reaction Plane**:

$\varepsilon\{std\}$, $\varepsilon\{4\}$

v_2 that are sensitive to anisotropy w.r.t. the

Participant Plane :

$v_2\{2\}$, $v_2\{EP\}$, $v_2\{uQ\}$ etc.

ε That are sensitive to anisotropy w.r.t. the

Participant Plane:

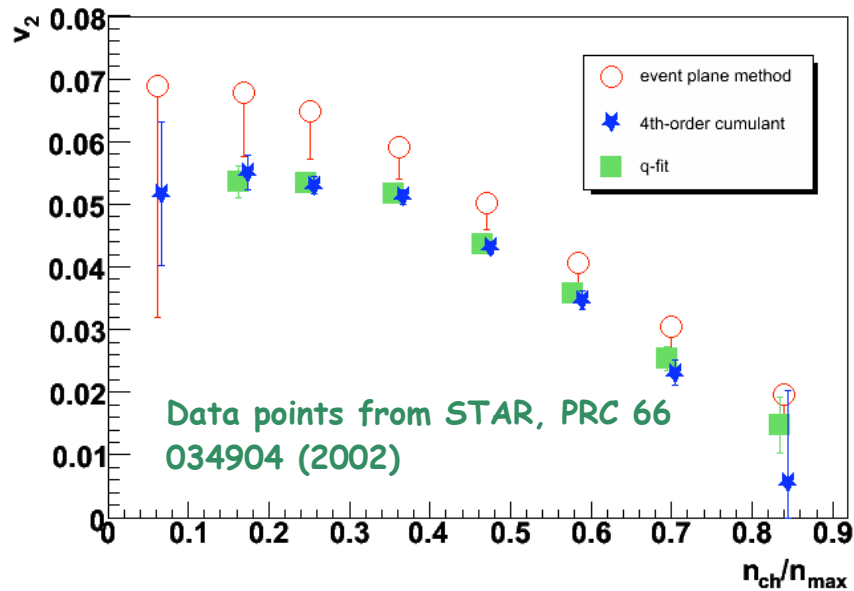
$\varepsilon\{part\}$ $\varepsilon\{2\}$

R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260

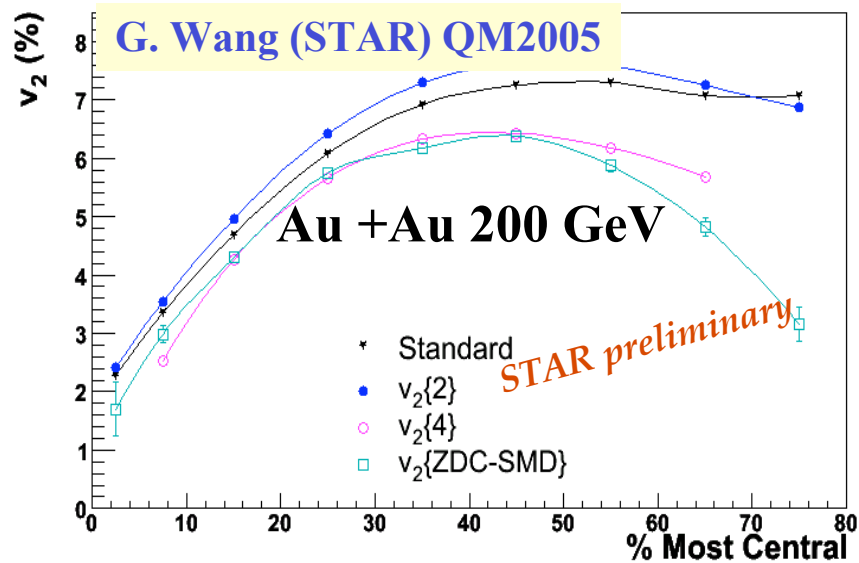
S.Voloshin, A.Poskanzer, A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537



v_2 methods



$$v_2\{4\}=v_2\{qDist\}$$

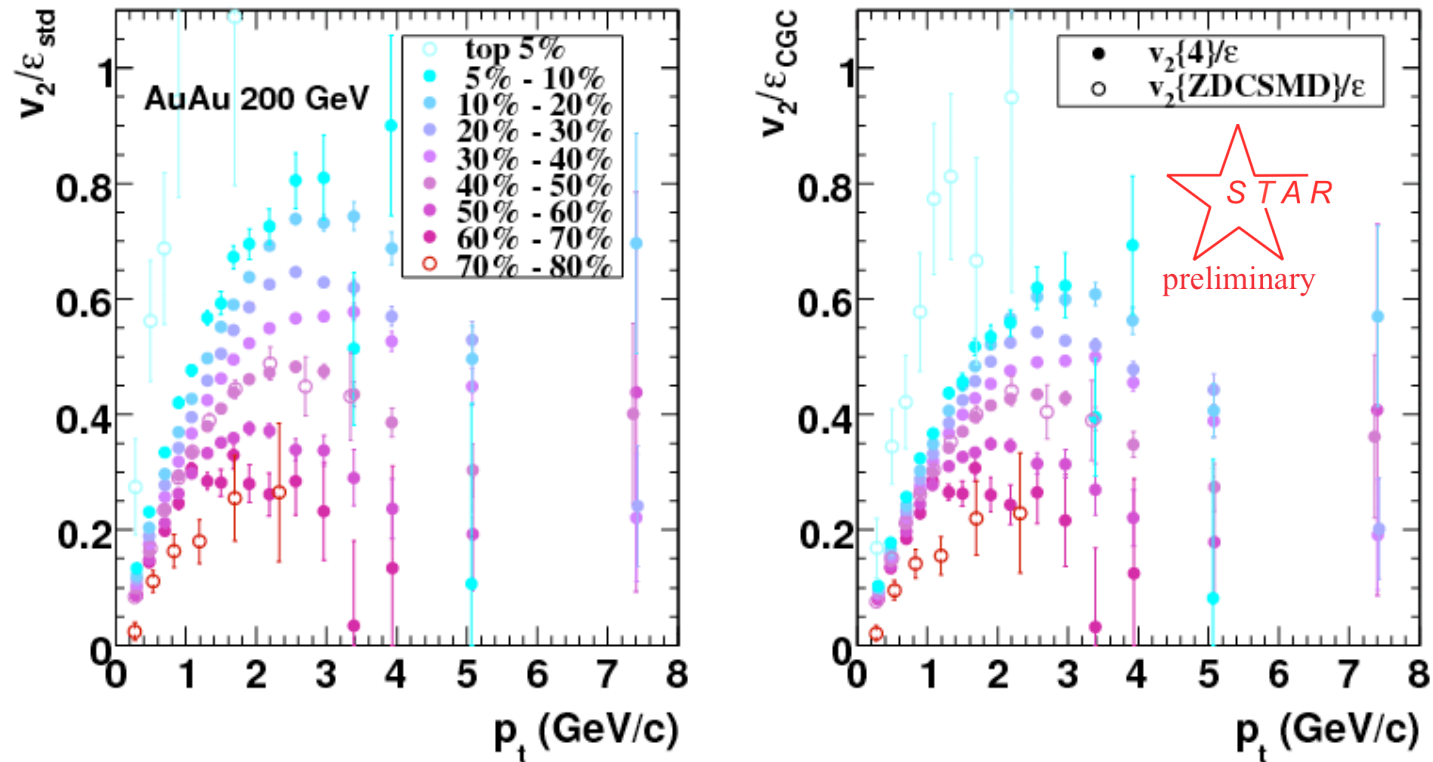


$$v_2\{4\}=v_2\{ZDC-SMD\}$$



Flow increases

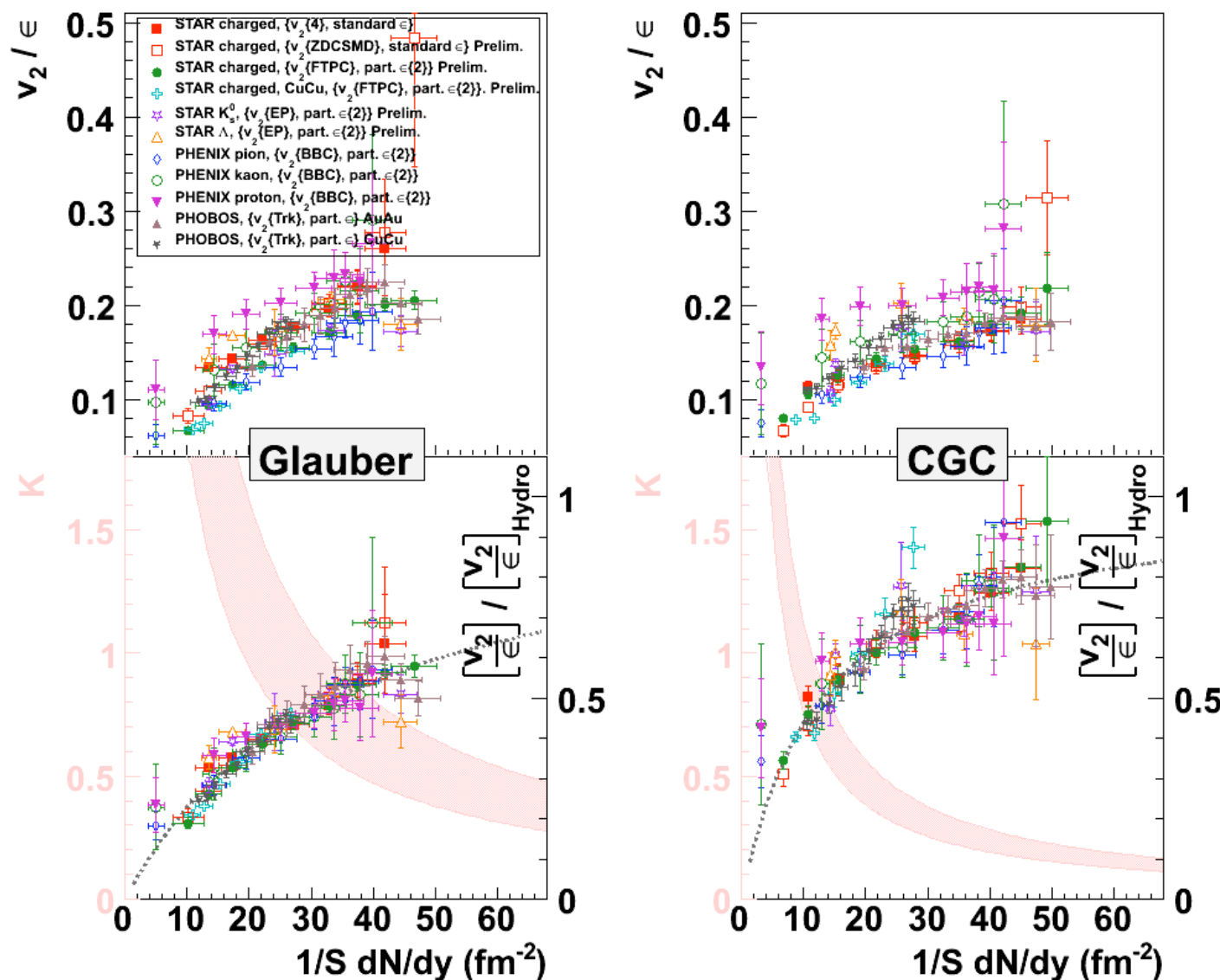
Y. Bai SQM08



The p_t where v_2/ϵ peaks increases with p_t - the applicable range
For hydrodynamics extends to large p_t in central collisions.
 v_2/ϵ for the CGC case sees hints of saturation.



How much deviation from ideal hydro ?



STAR Preliminary
data taken from
G.Wang, QM05
Y.Bai SQM08
H.Masui SQM08

PHENIX data points
taken from
nucl-ex/0604011

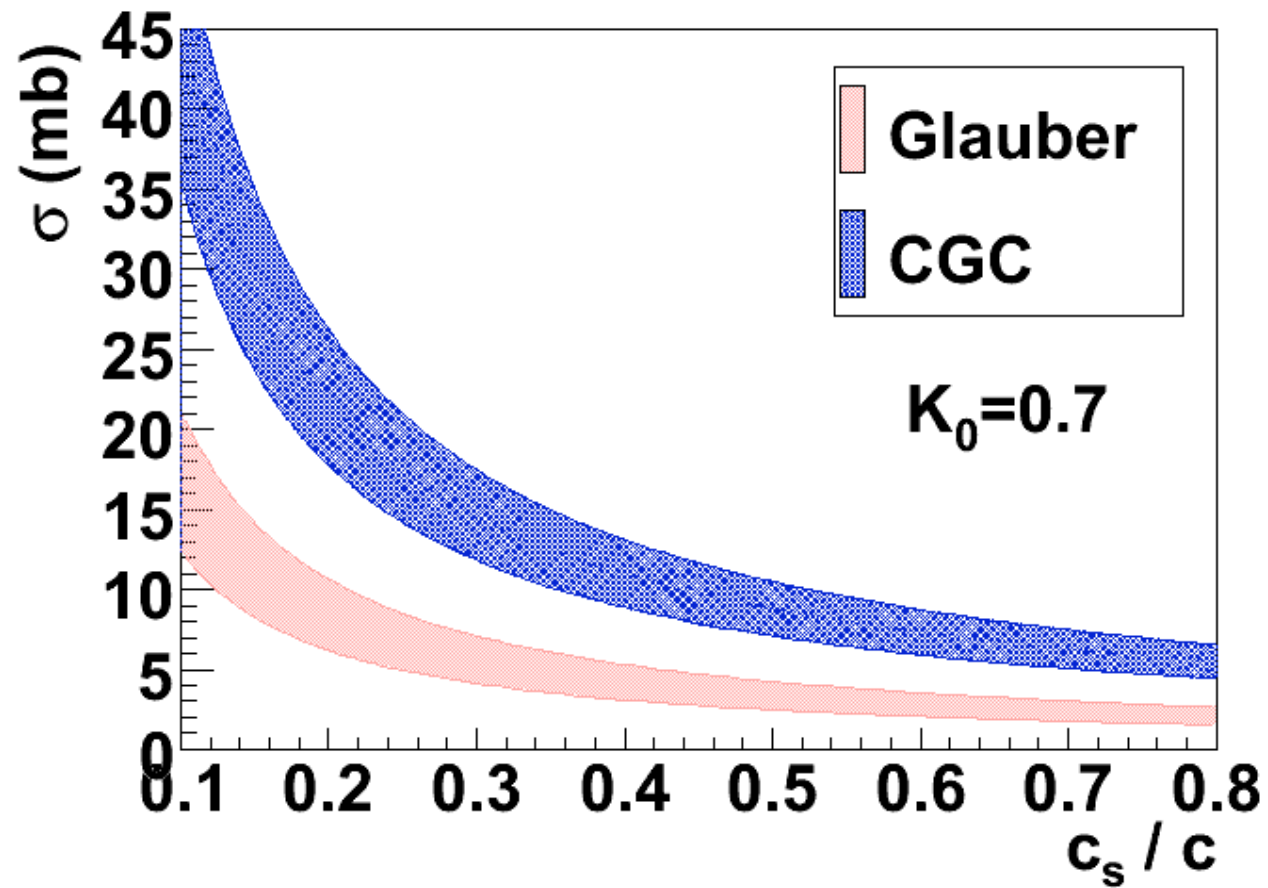
PHOBOS data points
taken from
PRC72 051901 (2005)
PRL 98 242302 (2007)

Fitting function from
Drescher, Dumitru,
Gombeaud, J.Ollitrault,
Phys. Rev. C76,
024905(2007)

CGC ϵ obtained from
A.Adil, H-J
Drescher, A.Dumitru,
A.Hayashigaki and
Y.Nara, Phys. Rev. C 74
044905 (2006)



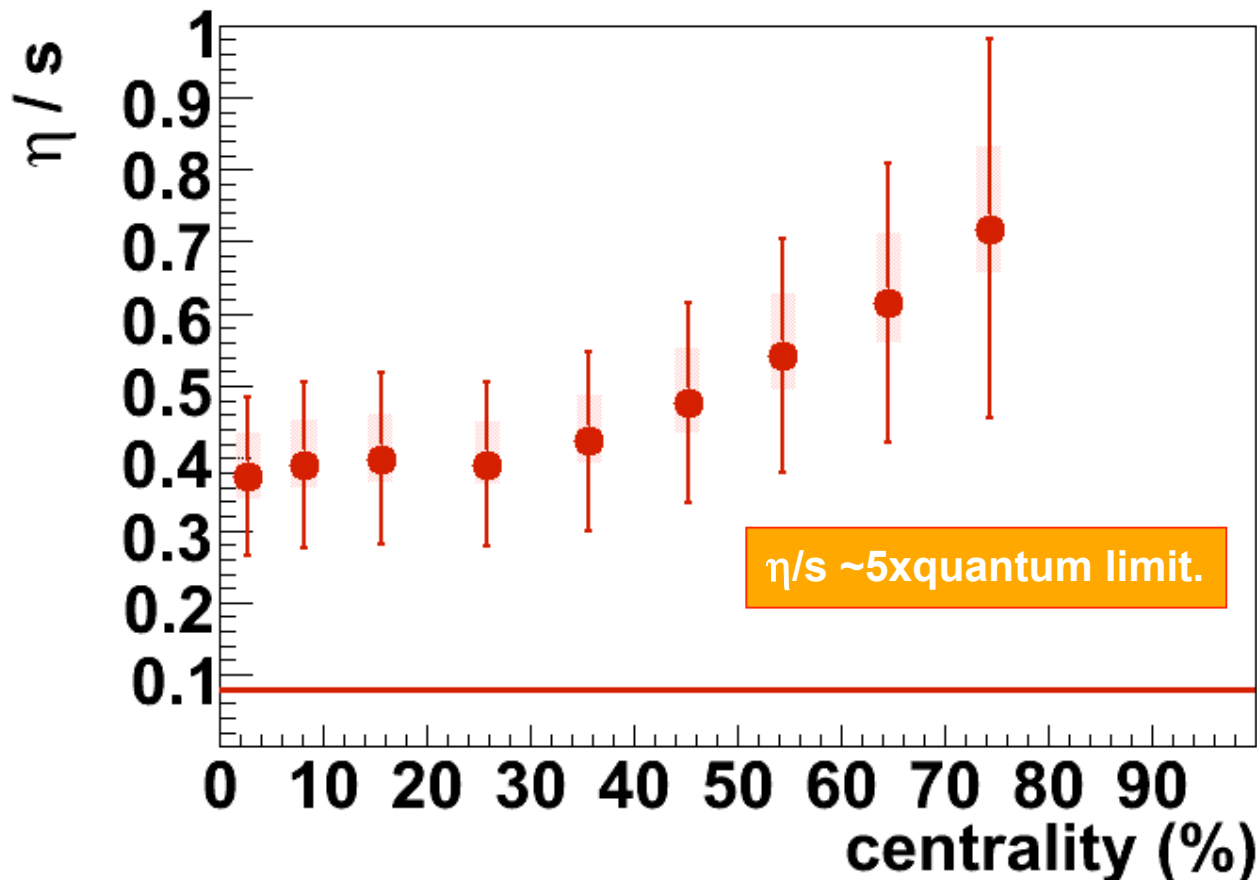
Constraint on EoS



$$\sigma\left(\frac{c_s}{c}\right)k_0 = \text{const.}$$



Constraint on η/s

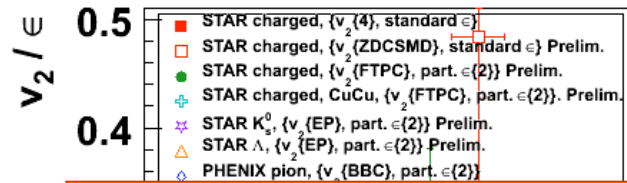


- Sys. error from uncertainties, correlated and uncorrelated, in v_2 , ϵ , S , and dN/dy are included in the fitting procedure.
- Uncertainty due to the sys. error of T and the choice of formula is represented by the light red box.
- Additional uncertainties will be discussed later.

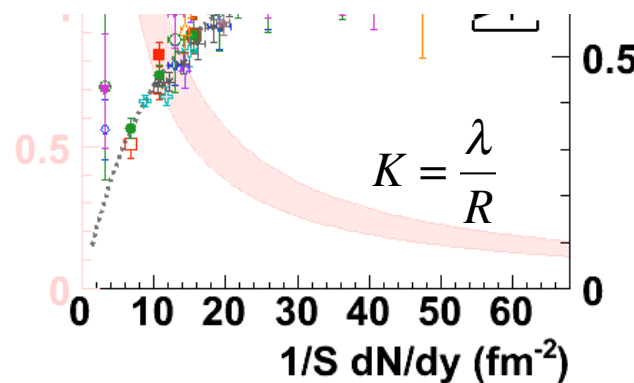
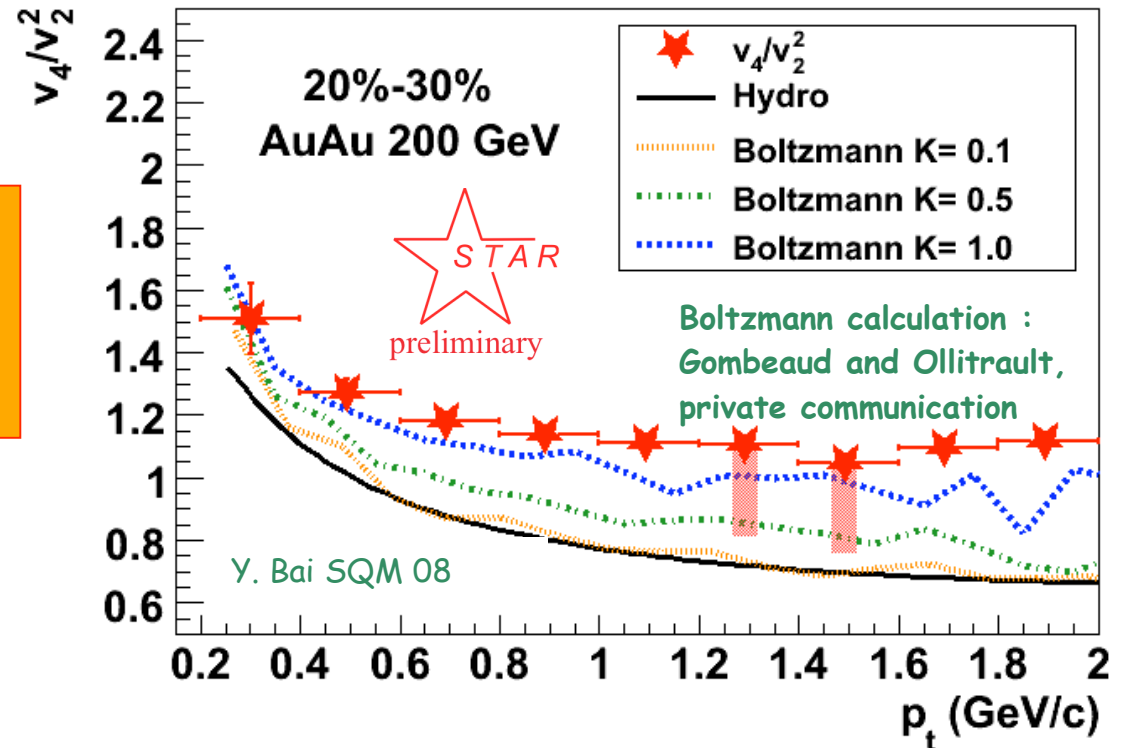
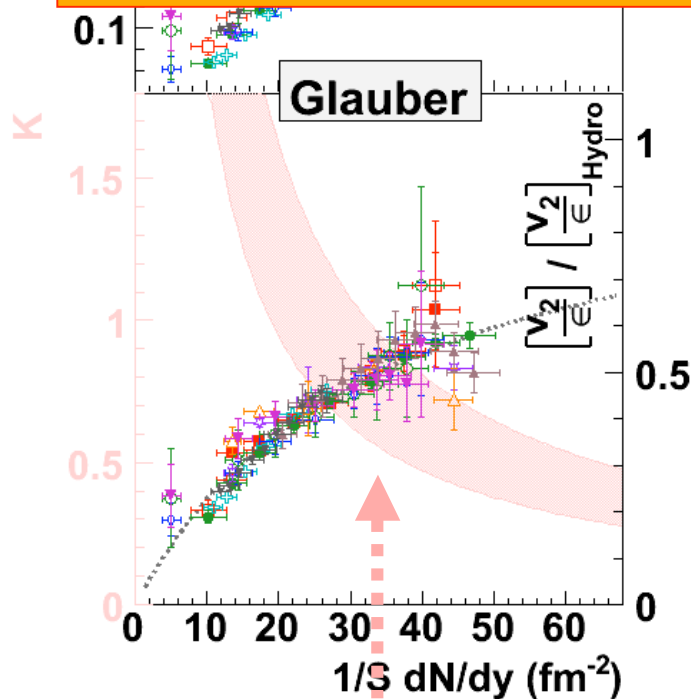
$$\frac{\eta}{s} = 0.316 \frac{\lambda T}{c} = 0.316 \frac{T}{c \sigma n} = 0.316 \frac{T}{c \sigma \frac{1}{S} \frac{dN}{dy} \cdot \frac{1}{R/(c_s/c)}} = 0.316 \frac{T}{\sigma c_s} \cdot \frac{1}{\frac{1}{SR} \frac{dN}{dy}}$$



A slightly different approach on Knudsen number



For centrality 20-30%,
Knudsen number needs to
be greater than 0.5 in order
to explain data



$K=0$: Ideal Hydro
 $K \gg 1$: Free Streaming



For the discussion

$$\frac{1}{K} \equiv \frac{R}{\lambda}$$

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct} \frac{1}{S} \frac{dN}{dy}$$

$$t \sim R / c_s$$

- Time independent K. Correction on effective λ . $\eta/s \downarrow$
- Correction on effective T. $\eta/s \uparrow$
- Uncertainty in K_0 (?). Needs a better answer from viscous Hydro calculation.
- ...

$$\frac{v_2}{\varepsilon} = \left[\frac{v_2}{\varepsilon} \right]_{hydro} \frac{1}{1 + K / K_0}$$

$$\frac{\eta}{s} = 0.316 \frac{\lambda T}{c} = 0.316 \frac{T}{c \sigma n} = 0.316 \frac{T}{c \sigma \frac{1}{S} \frac{dN}{dy} \cdot \frac{1}{R / (c_s / c)}} = 0.316 \frac{T}{\sigma c_s} \cdot \frac{1}{\frac{1}{SR} \frac{dN}{dy}}$$



Summary

- v_2/ϵ , if examined with transport motivated formula with certain assumptions, approaches Hydro limit in a similar way for different particle species.
- In central AuAu collisions, with Glauber initial condition, v_2/ϵ is 2--30% away from Hydro limit. This conclusion is independent of PID, initial conditions, and choice of $\{v_2, \epsilon\}$ pairs.
- Knudsen number extracted from fitting v_2/ϵ vs. $1/S$ dN/dy , as well as that extracted from v_4/v_2^2 , stays finite for central collisions - not as zero as required by ideal hydrodynamics.
- Constraint on EoS obtained.
- For the first time the centrality dependence of η/s is presented.